# Robustica Case Study: Snap fastener Design for Disassembly and Recyclability

## Background

Snap fasteners provide a quick and easy method of assembly. However, with the recent increase in demand for recyclability, products must now be designed for disassembly as well as assembly. This demand has been made more acute by the introduction of 'take back' programs around the world. Under such schemes, the responsibility of the manufacturer is extended to the disposal of their products. This in turn means that the cost of disposal falls on the manufacturer. With increases in fees for traditional disposal (dumping), recycling becomes a more economic option. To ensure the economics, it is important that all products be as easy to recycle as possible. Typically, snap fasteners are designed to snap together with relative ease and hold securely together for product quality. This second characteristic makes disassembly, and recycling, difficult and more costly. Still, the snap fasteners are ideal for cost effective assembly.

Faced with this situation, a particular manufacture of plastic appliances wanted to find a method of attachment that would:

- allow easy assembly of plastic parts
- be easy to disassemble
- still provide sufficient attachment between parts

One potential solution considered by the design engineers was a snap fastener that can be detached easily with a dedicated tool that provides sufficient leverage. Such a fastener would allow easy assembly and not come apart during typical customer use, but still allow for easy recycling.

## The problem

The design engineers need to design a snap fastener that is easy to assemble (low push in force) and that would only come apart easily when the right tool was used (an extraction force that is not too high nor too low).

## The solution

The proposed design for the snap fastener is shown in figure 1.

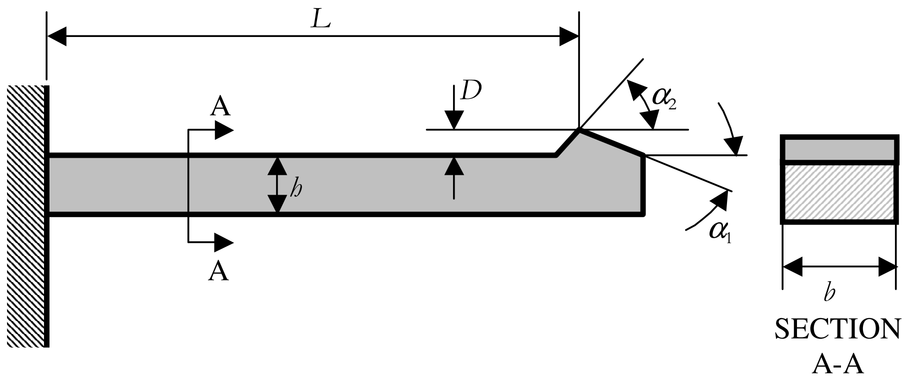


**Figure 1. A snap fastener**

When a snap fastener is pushed into its mating lug, shown in blue in figure 2, it deflects a distance $D$ before 'snapping' into the lug.

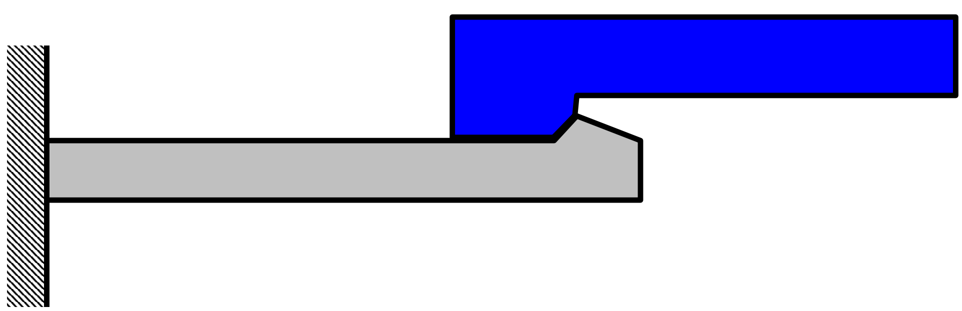

**Figure 2. A snap fastener and its mating lug**

The axial force required to overcome the deflection is a function of the deflection force and the angle of the face over which the lug slides. Therefore, by having the forward and backward faces at different angles, $\alpha_1$ and $\alpha_2$ respectively, the insertion force $f_i$ and the extraction force $f_e$ are also different.

To make the solution work, the design must be such that the following are met:

- The insertion force is not so great that it reduces manufacturability
- The extraction force is not so great that it inhibits disassembly
- The extraction force is as great as possible, and parts stay assembled

From the above criteria, it is concluded that a snap fastener must be designed with:

- a low probability of a high insertion force, so that parts can be easily assembled
- an extraction force not exceeding the maximum permissible (for disassembly) extraction force, but high enough so that parts are most unlikely to come apart in use

To achieve this, the design of the snap fastener in figure 1 will be modeled analytically, the random variability of the inputs will be ascertained, and then the system will be modeled in Excel. Finally, Robustica will be used to find the optimum design.

# Modeling the snap fastener

## Forces

The deflection force is found using beam deflection theory and treating the snap fastener as a cantilever[1]. By applying this theory to the snap fastener in figure 1, the formula for deflection force $P$ is:

$$P = \frac{3\,D\,E\,I}{L^3}$$

Where:

$I$ is the moment area of inertia of the cross section of the cantilever $b\,h^3 / 12$

$E$ is the effective modulus of elasticity

The axial forces are related to the deflection force by the angle of the respective face and the coefficient of friction $\mu$ between the snap fastener and the lug.

$$f_i = P\,\frac{\mu + \tan\alpha_1}{1 - \mu\tan\alpha_1}$$

$$f_e = P\,\frac{\mu + \tan\alpha_2}{1 - \mu\tan\alpha_2}$$

## Failure modes

For easy assembly the insertion force cannot be greater than 50 N. This force is the maximum that the assembly workers can apply using the assembly equipment

[1] **Hamrock et al. 1999** 'Fundamental of Machine Elements' WCB/McGraw-Hill.

and still maintain the required production rate without the risk of any related injury. If the force were above this level on each part, then the estimated cost due to injury per part would be \$5. If the force were below 40 N, then the operator would not receive sufficient feedback to determine that assembly had been successful. Under these conditions the operator would need to take extra action to confirm assembly. This would cost \$5 each time.

Ideally, the extraction force will be less than the maximum extraction force of 200N. This is the maximum force that can be easily applied using the disassembly equipment. A greater force would require excessive effort and time, and incur a cost of \$10 for disposal via other means.

The appliance for which this snap fitting is to be used would be subjected to a maximum separating force of 100 N during normal use. If the extraction force were below this separating force then the appliance would fail with an estimated cost of \$50 (this includes the cost of the customer's time and damage to reputation of the appliance's brand and image).

While it is not explicitly stated, obviously the plastic cannot be allowed to yield. The most significant stress in a cantilever is bending stress $\sigma_b$ and this cannot be allowed to exceed the yield stress $\sigma_y$.. This failure mode is expressed mathematically below.

$$SF = \frac{\sigma_b}{\sigma_y} \geq 1$$

Where:
*SF* is the safety factor

At worst, this will cause a failure that will not be spotted in the factory, and the appliance will leave with a snap fitting of reduced strength. This will result in the cover of the appliance coming apart in regular use. As discussed above such a failure mode will in incur a cost of \$100, and this is the cost of failure associated with *SF* being below 1.

The bending stress can also be derived from beam bending theory.

$$\sigma_b = \frac{M\ y}{I} = \frac{P\ L\ \frac{h}{2}}{\frac{b\ h^3}{12}} = \frac{6\ P\ L}{b\ h^2}$$

These formulae were easily entered into Excel as shown in the accompanying worksheet file 'Snap_fastener_example.xls'. However, before robustification can be performed, the nature of the random variability for each of the input variables must be known.

# Random variability of the input variables

## Linear dimensions

The manufacturer of the parts, a molding company, indicated that the standard tolerance for molded plastic dimensions *t* is ± 0.1mm. This will be assumed for all of the dimensions of the snap fastener unless special action is taken to reduce the tolerance. Further, it will be assumed that the distribution for these dimensions is a Normal distribution. The standard deviation will be set at 0.1/3 mm, this is equivalent to assuming that 99.7% of the time the dimension is within the tolerance.

Further discussions regarding the tolerance were had between the design engineers and the production engineer of the molding company. The production engineer said that she could change the molding operation to reduce the tolerance range to ± 0.07mm. However, this would increase production time, and the cost would increase by \$0.2 per part. The design engineers decided to consider this as a possibility. Further, a linear relationship between cost *C* and the tolerance *t* was assumed over the range of above tolerance values.

$$\begin{aligned}
&C = m\,t + c \\
&(0.1, 0) \Rightarrow 0 = m \times 0.1 + c \Rightarrow c = 0 - 0.1\,m && 1 \\
&(0.07, 0.2) \Rightarrow 0.2 = m \times 0.07 + c \Rightarrow c = 0.2 - 0.07\,m && 2 \\
&2 - 1 \Rightarrow 0 = 0.2 + 0.03\,m \Rightarrow m = -6.67 && 3 \\
&3 \to 1 \Rightarrow c = 0 - 0.1 \times (-6.67) = 0.667 \\
&\therefore C = -6.67\,t + 0.667
\end{aligned}$$

## Geometry

The molding company also indicated that various aspects of the geometry would not change. This is because over small areas the mold temperature and pressure remain relatively constant; therefore, the thermal and flow related changes in the linear dimensions of the features within that area are the same. For this reason, the angle dimensions $\alpha_1$ and $\alpha_2$ were treated as constants. Based on further advice form the molder, it was assumed that the toolmaker would be able to modify the mold if necessary to ensure that the angles are equal to those specified. Therefore, no randomness in the tool making needed to be taken into account either.

For similar reasons, the ratio between some of the linear dimensions will also remain constant. For this case, the ratio between *h* and *b* would remain constant. Therefore, the ratio of the width to the height *B* will also be treated as a constant.

## Friction

The coefficient of friction is given a Lognormal distribution with a mean of 0.6 and a standard deviation of 0.021. This is based on the range of possible values given for the coefficient of friction between polycarbonate and itself[2].

## Yield strength and Young's modulus

The molding company said that the Polycarbonate sourced would have a yield strength between 60 and 70 MPa and a Young's modulus of elasticity between 2 and 2.5 GPa[3]. With no other indication of the nature of variability that would be expected for these variables, a Uniform distribution was used. Therefore, the standard deviation was equal to the permissible range divided by the square root of 12.

The design engineers raised the issue of correlation between the two material properties. While the production engineer from the molding company was uncertain, she suspected that the stiffer polycarbonate would also be the stronger.

[2] **Hamrock et al. 1999** 'Fundamental of Machine Elements' WCB/McGraw-Hill p716.
[3] Young's modulus is not entirely applicable to a polymer, but it was used here for convenience.

Therefore, it was assumed that the yield strength would be a function of the modulus as derived below[4]:

$$\begin{aligned}
&\sigma_y = m\,E + c \\
&(2000,\,60) \Rightarrow 60 = m \times 2000 + c \Rightarrow c = 60 - 2000\,m && 1 \\
&(2500,\,70) \Rightarrow 70 = m \times 2500 + c \Rightarrow c = 70 - 2500\,m && 2 \\
&2 - 1 \Rightarrow 0 = 10 - 500\,m \Rightarrow m = 0.02 && 3 \\
&3 \rightarrow 1 \Rightarrow c = 60 - 2000 \times 0.02 = 20 \\
&\therefore \sigma_y = 0.02\,E + 20
\end{aligned}$$

The above equation assumes a linear relationship between yield strength and Young's modulus. The design engineers realized that this may not be correct, but it was all that was afforded by the information at hand.

## Robustification

As mentioned above these equations were entered into an Excel spreadsheet to create a model, which can be seen in 'Snap_fastener_example.xls'. The initial input values that are shown in the spreadsheet were chosen because they provide suitable assembly and extraction forces while keeping the safety factor above 1.

The limits for each of the input variables are shown below in table 1. The limits were set so as to prevent interference with other features of the appliance and to allow for the easy molding of the parts.

| Variable | Symbol | Lower limit | Upper limit | Units |
|---|---|---|---|---|
| Deflection | $D$ | 0.001 | 0.005 | m |
| Cantilever length | $L$ | 0.01 | 0.05 | m |
| Cantilever height | $h$ | 0.0015 | 0.01 | m |
| Forward face angle | $\alpha_1$ | 0.1 | 1 | rad (1) |
| Backward face angle | $\alpha_2$ | 0.1 | 1 | rad (1) |
| Width height ratio | $B$ | 0.5 | 3 | 1 |

**Table 1. Input variables and their limits**

Along with the limits shown in table 1, the region investigated by Robustica during robustification will also include tolerance values between ±0.07mm and ±0.1mm.

[4] This function assumes total correlation, but because this will increase the predicted variability it is considered the safest assumption to make.

### Pre Robustification

Initially, the mean and the standard deviation for each of the output variables were as shown in table 2. Also, the contributions to this variability from the input variables were as shown in figure 3.

| Name | Symbol | Nominal | Mean | Standard deviation |
|---|---|---|---|---|
| Insertion force | *fi* | 45.81 | 45.93 | 4.27 |
| Extraction force | *fe* | 145.59 | 150.24 | 32.76 |
| Safety factor | *SF* | 3.38 | 3.39 | 0.095 |

**Table 2. Initial output variables**

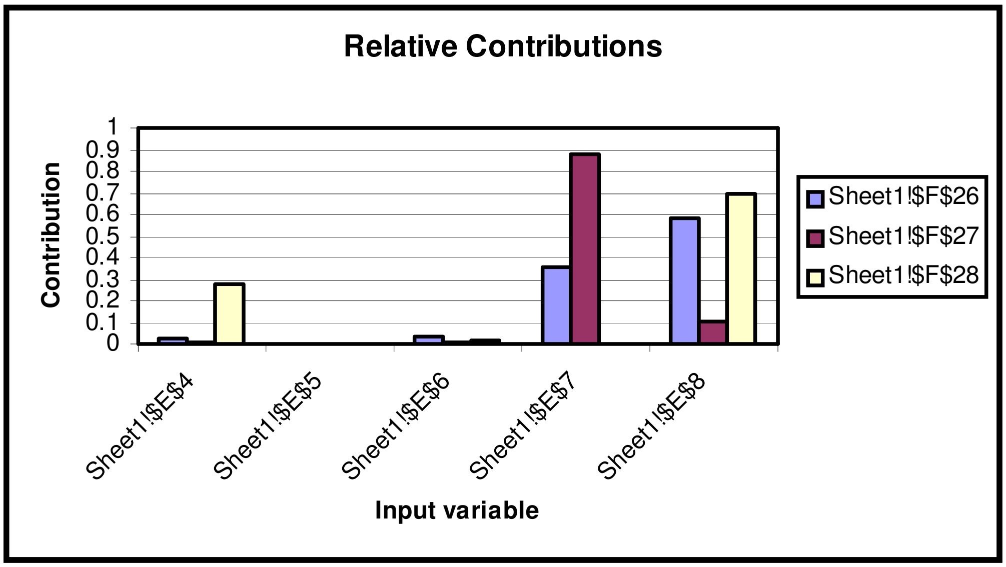


**Figure 3. Initial contributions chart**

From table 2 it can be determined that the major concern with the initial design will be the random variability in the extraction force. In addition, from figure 3 it can be seen that the major sources of variability are the coefficient of friction and the modulus of elasticity. Because the major sources of random variability are basically noise variables (or parameters to be more accurate), the optimised design will need to reduce the sensitivity to variables that are practically beyond the control of the designer. Therefore, any reduction in the random variability of the physical dimensions will likely provide little gain in quality, and it is expected that the optimised design will not require any increase in manufacturing expense.

## Post Robustification

The optimum values for the input variables found by Robustica after half an hour are shown in table 3. The mean and the standard deviation for each of the output variables after optimisation are shown in table 4. Also, the contributions to this variability from the input variables are shown in figure 4.

| Variable | Symbol | Lower limit | Units |
|---|---|---|---|
| Deflection | $D$ | 0.0042 | m |
| Cantilever length | $L$ | 0.042 | m |
| Cantilever height | $h$ | 0.0074 | m |
| Forward face angle | $\alpha_1$ | 0.12 | rad (1) |
| Backward face angle | $\alpha_2$ | 0.64 | rad (1) |
| Width height ratio | $B$ | 0.67 | 1 |

**Table 3. Final input variables**

| Name | Symbol | Nominal | Mean | Standard deviation |
|---|---|---|---|---|
| Insertion force | $fi$ | 44.70 | 44.71 | 3.61 |
| Extraction force | $fe$ | 138.87 | 139.05 | 11.91 |
| Safety factor | $SF$ | 1.12 | 1.12 | 0.030 |

**Table 4. Final output variables**

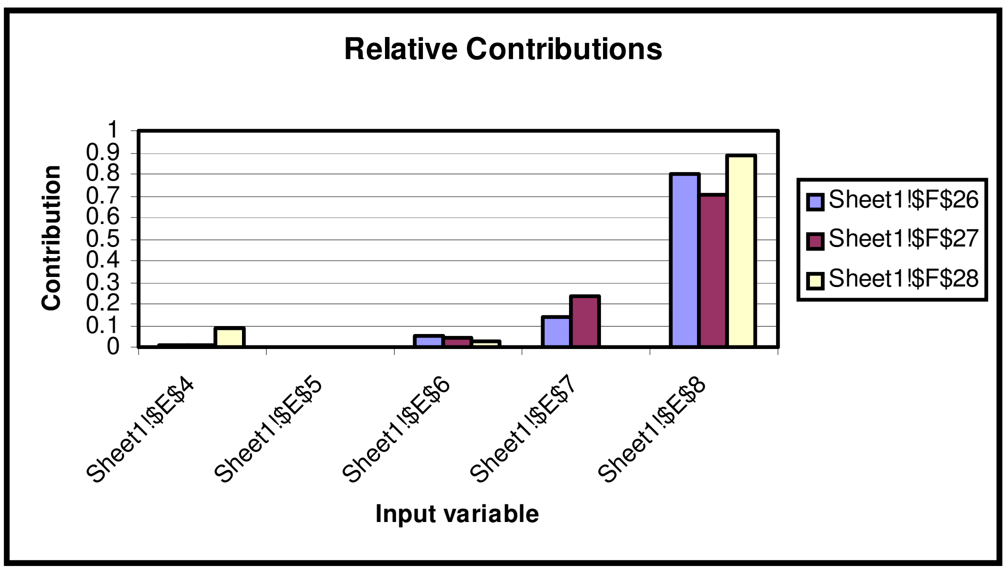


**Figure 3. Final contributions chart**

It was also found that the initial tolerance ratting of ± 0.1mm for the feature dimensions was optimum, as expected. The reader can enter the input variables of table 3 to confirm the results if they wish. In either case, it can be seen that the variability of the outputs has been reduced and that the major source of variability is now only the modulus of elasticity. Therefore, the cost of failure has been reduced prior to manufacture commencing. In addition, the design engineers also know that if further gains are required, focus should be directed toward the control of material properties and not the control of the physical dimensions.

## Summary

The above case showed that the creation of the model can be achieved with relative ease when the system under consideration is well documented in available texts. Nevertheless, the designer or modeler still needs to communicate with others so that they can acquire an accurate understanding of the random variability that can be expected. Once this is done, the application of Robustica is relatively easy.

Two major issues regarding the application of probabilistic methods to manufactured products were also highlighted by this case. First was that there is an advantage to applying these methods prior to manufacture. Consider the case where only the nominal values produced by the model had been used to find a suitable design. It would not have been until manufacturing had commenced that the cost associated with random variability would have been known. Second, by considering the sources of variability before taking any other action, wasted effort can be reduced. The design engineers now know that they should focus on elasticity and not the tolerances of the physical dimensions if more improvements are needed.